647510 **MSC1PhyC003x** **Seat No :________**

M.Sc.(Phy.) Semester - 1 (*CBCS*) Examination

Nov/Dec - 2022 [NEW COURSE]

Quantum Mechanics-I(Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q - 1: Answer in brief any <u>SEVEN</u> **14**

(a) Why the positive sign in the exponent is avoided in the solution of harmonic oscillator?

(b) Prove that $\boldsymbol{a}\boldsymbol{a}^{\dagger} = \frac{\boldsymbol{H}}{\hbar \boldsymbol{w}} - \frac{\mathbf{1}}{\mathbf{2}}$

(c) Prove that $[\mathbf{H}, \boldsymbol{a}^{\dagger}] = \hbar\boldsymbol{\omega}\boldsymbol{a}^{\dagger}$

(d) Prove that $[J_z, J_+] = \hbar J_+$

(e) Give the relations of Rectangular and Spherical Polar Coordinates.

(f) Define trial wave function. How it is selected?

(g) What is the main application of Variation Principle?

(h) WKB approximation is known as semi-classical approximation. Why?

(i) Why Fermi – Golden rule is known as Golden? Write down the application of it.

(j) What is the application of time dependent perturbation?

Q - 2: Answer the following questions: Any <u>TWO</u> out of THREE **14**

(a) Solve the below given equation for one dimensional harmonic oscillator using power series technique –

$$\frac{d^2h}{d\xi^2} - 2\xi\frac{dh}{d\xi} + h(\epsilon - 1) = 0$$

(b) Discuss the orthogonality of Hermite Polynomial and show that –

$$I_{m,n} = \int_{-\infty}^{+\infty} u_m(\xi)\, u_n(\xi) d\xi = 0 \quad \text{if } m \neq n$$

(c) Discuss Harmonic Oscillator Energy Spectrum.

Q - 3: Answer the following questions: <u>BOTH</u> ARE COMPULSORY **14**

(a) Derive $\vec{L}^2$ in the spherical polar coordinates as -

$$\vec{L}^2 = -\hbar^2 \left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\emptyset^2}\right]$$

(b) Using the concept of raising and lowering operators for angular momentum, obtain the below given relations,

(i) $\lambda_J - m_{max}(m_{max} + 1)\hbar^2 = 0$

(ii) $\lambda_J - m_{min}(m_{min} - 1)\hbar^2 = 0$

<u>OR</u>

Q – 3: Answer the following questions: <u>BOTH</u> ARE COMPULSORY **14**

(a) Write Schrödinger equation in polar coordinates and obtain the radial equation for r - only.

(b) For attractive Coulomb potential, solve the Schrödinger radial equation and obtain the energy eigen values as –

$$E_n = -\frac{mz^2e^4}{2\hbar^2 \mathrm{n}^2}$$

Q - 4: Answer the following questions: Any <u>TWO</u> out of THREE **14**

(a) Discuss Variational Method in detail.

(b) If any anharmonic oscillator is having the Hamiltonian as -

$$\mathrm{H} = \frac{p2}{2m} + \frac{1}{2}kx^2 + \mathrm{a}x^3 + \mathrm{b}x^4 ,$$

then prove that the perturbation shifts the ground state energy to higher level.

(c) What is meant by time dependent perturbation? Obtain the below given general formulation –

$$\left[\mathrm{i}\hbar\frac{\mathrm{dc_m(t)}}{\mathrm{dt}}\right] = \sum_{\mathrm{n}} \mathrm{c_n(t)} e^{\frac{\mathrm{i(E_m - E_n)t}}{\hbar}} \langle \Phi_{\mathrm{m}} | \mathrm{H}_1 | \Phi_{\mathrm{n}} \rangle$$

Q - 5: Write any <u>TWO</u> short-notes on: **14**

(a) The Raising, Lowering and Number Operators

(b) Spherical Harmonics

(c) WKB Approximation

(d) Hydrogen atom and its energy eigen values

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